A Viewpoint on Construction of Networked Model of Event-triggered Hybrid Dynamic Games

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***Abstract*—This paper studies the modeling problem of event-triggered networked hybrid dynamic games (HDGs). By considering the influence of an event-triggering mechanism, the evolution process of dynamic games with hybrid characteristics is analyzed, and we point out the complexity and technical difficulties in the analysis of such a game problem. From the perspective of network science, we give a viewpoint on network-based modeling of event-triggered HDGs. On the basis of the state-space model with established, we first give the normal form of HDGs by a seven-tuple. We then establish the directed dynamic network model of HDGs for the first time, involving a graph-based tree structure form , which can well describe the distinctive features of the continuous-time and discrete-event dynamic game process on both sides, and has great advantages in evolutionary analysis. An example of the evolution of event-triggered HDGs show the innovation of the proposed model.**

***Index Terms*—Hybrid dynamic game, Event-triggering mechanism, Graph-based tree structure form , Directed dynamic network**

I. Introduction

The study of game theory has seen many great achievements [1]–[3]. In recent years, hybrid dynamic games (HDGs) [4] have attracted much attention. These originate from chess games [5], [6] and combat action [7], and describe a multi-stage hybrid dynamic decision-making process with the interaction of discrete event triggering and continuous control between two non-cooperative agents. Compared with traditional continuous dynamic games [2], the evolution process of HDGs is affected by an event-triggering mechanism that causes structural changes to the system at discrete time points and shows hybrid dynamic characteristics. Although the development of computer games has focused on algorithmic solutions of complex game problems [6], [8]–[11], modeling and analysis of such a dynamic game system presents a significant challenge owing to the complexity of its intrinsic hybrid evolution process.

Over the past decade, Xu and Shi [4] analyzed the air fight evolution process, proposed the basic modeling framework of HDGs for the first time, and addressed a series of key issues on modeling, analysis, and control. How to establish a system model to better describe the internal dynamic evolution has become the primary problem to be solved in the study of HDGs. Platzer et al. [12], [13] introduced a class of differential hybrid game models combining differential and hybrid games. Gromov and Gromova [14] presented a systematic application of a hybrid system framework to differential game models. Chen et al. [15], [16] used the Lanchester equation to establish a state-space model of warfare hybrid dynamic games. From the perspective of discrete event evolution analysis, many important models, such as event dynamic games [17], event-triggered discrete-time zero-sum games [18], and logical state space models of finite games [19], are established considering the impact of event triggering. However, the above results are not perfect in the analysis of the internal evolution rules and system structure characteristics of HDGs. There is still a lack of a theoretical basis and effective characterization for complex HDGs with large strategy sets and multiple stages.

In the late 20th century, the development of network science [20], [21] and the deepening of evolutionary games [21], [22] to asymmetric games provided a new direction for the study of HDGs. Due to the increase of the system scale and complexity, HDGs presents the characteristics of dynamic network evolution, which not only contains the temporal evolution of continuous states, but emerges multi-layer features with the update of event-triggered mechanisms. Many network game models for continuous dynamic evolution have been proposed. For example, Li et al. [23] established the evolution model of an attack-defense game from the perspective of network science, and a multilayered attack-defense game on networks was proposed in [24]. Various attacker and defender strategies within a dynamic game on network topology were evaluated in [25]. A multi-layer network formation [26] was

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considered to investigate the noncooperative dynamic game problem [27]. Zhang et al. [28] provided a thorough analysis of a temporal network game model, in which individuals play a divide-and-conquer game with their neighbors. Yeh et al. [29] analyzed a multistage temporal network model for 2048-like games. Logical dynamic networks [30] have become important models of network evolutionary games in an event domain. Cheng et al. [31] applied a semi-tensor product method to the evolution analysis of finite games and networked evolutionary games, and provided a matrix representation of an event-triggered dynamic game model [32], [33]. Event trees also play a role in the evolution of complex dynamic games, including constrained linear-quadratic dynamic games [34], [35], cooperative dynamic games [36], and minimum cost spanning tree games [37]. Based on the above literature review, we can easily conclude that the above network models provide support for the network-based modeling of HDGs.

Inspired by the above analysis and literature review, we present a novel viewpoint on the network-based modeling of event-triggered HDGs. Based on the establishment of the normal form of HDGs by a seven-tuple, we provide the directed dynamic network evolution model for the first time, which can well describe the distinctive features of the continuous-time and discrete-event dynamic game process on both sides. An example demonstrates the innovation of the model.

II. Process Analysis and State-space Model

Let X and Y be two noncooperative players, with E_x and E_y as respective event-triggered move sets. U and V are continuous control strategies on both sides. Fig. 1(a) shows the evolution of a HDGs between two non-cooperative players X and Y. All event tactics, i.e., $E_x = \{E_{x1}, E_{x2}, \cdots, E_{xn}\}$ and $E_y = \{E_{y1}, E_{y2}, \cdots, E_{yn}\}$, are triggered at a discrete time instant, which cause structural changes of the continuous-time dynamic game process. Once all event tactics are determined, every hybrid dynamic sub-process will become differential game processes, and the continuous control strategies $U = \{U_1, U_2, \cdots, U_n\}$, and $V = \{V_1, V_2, \cdots, V_n\}$ are designed. Fig. 2(b) shows the interaction between a continuous-time process and a discrete event dynamic process, where S_0 is the initial situation and S_F the final situation. In the interplay between (U, V) and (E_x, E_y) from which players can choose, the situation sets are $S_X = \{S_{x1}, S_{x2}, \cdots, S_{xn}\}$ and $S_Y = \{S_{y1}, S_{y2}, \cdots, S_{yn}\}$. First, we give the general state-space model of HDGs as

$$\begin{cases} \dot{x} = f(x, y, E_x, U, \alpha, t), \\ \dot{y} = g(x, y, E_y, V, \beta, t), \end{cases} \tag{1}$$

where $\dot{x} < 0$ and $\dot{y} < 0$ and x and y are the state vectors, α and β are the non-negative attrition coefficients for the opposite sides, and $t \in [0, T]$, T is the terminal time and $x(T) > 0$ and $Y(T) > 0$. Let the objective function be defined as

$$\begin{aligned} &J(E_x, E_y, U, V) = \\ &\Phi(x(T), y(T)) + \textstyle\int_0^T h(x, y, E_x, E_y, U, V)dt, \end{aligned} \tag{2}$$

where $\Phi(x(T), y(T))$ is the terminal continuous function and $h(x, y, E_x, E_y, U, V)$ the continuous vector function. Clearly, the main goal of a HDGs is to design the optimal strategies (U^*, V^*) and the best event tactics (E_x^*, E_y^*), such that

$$\begin{aligned} &J(E_x^*, E_y, U^*, V) \geq J(E_x^*, E_y^*, U^*, V^*) \\ &\geq J(E_x, E_y^*, U, V^*). \end{aligned} \tag{3}$$

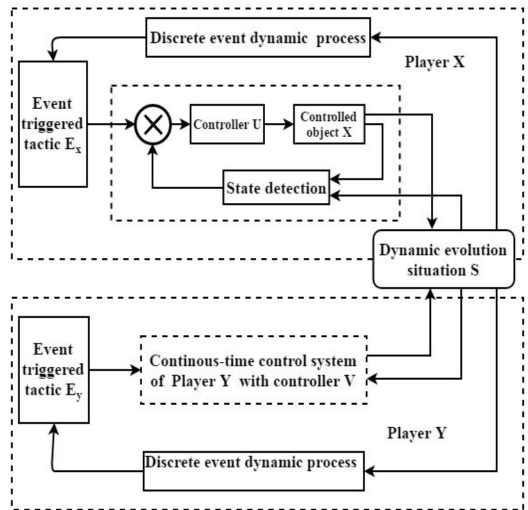

Fig. 1. Evolution process of hybrid dynamic games.

As mentioned in Ref. [15], $X = \{X_1, X_2\}$ and $Y = \{Y_1, Y_2\}$, and 16 optional schemes are obtained at each stage. When relatively few stages (one or two stages) are involved, their advantages on such a HDGs are demonstrated, as in (1) and (2). Nevertheless, when $X = \{X_1, X_2, \cdots, X_n\}$ and $Y = \{Y_1, Y_2, \cdots, Y_m\}$ are satisfied, $m^n n^m$ kinds of tactics decide which fight situation is taken in a stage. When a multi-stage HDGs process is also considered, the number of selectable strategies is significantly increased, which causes great difficulties in achieving strategy optimization.

III. Networked Modeling of HDGs

We first give the normal form of a HDGs by a seven-tuple as follows,

$$G = (P, S, C, E, \Sigma, R, F), \tag{4}$$

where $P = \{X, Y\}$ is a finite set of players, and each player has a certain number of units. S is a situational set including all evolutionary situations, and $S = \{S_0, S_X, S_Y, S_F\}$. C is the pure continuous control strategy set and $C = \{U, V\}$. E is the event tactics set, $E = \{E_X, E_Y\}$. Σ is the situation transition function about C and E, we label $\Sigma : S \times (E \times C) \to S$, which indicates the structural changes of the HDGS under the influence of C and E. R is the game rule including the player's actions sequence, information sets, and all pre-set game modes. F is the payoff function.

Remark 1. Σ transforms the system structure from one situation to the next, i.e., $S \times E \to S'$, where S' is the situation set that belongs to the evolution of continuous states, $S' \subseteq S$. Once E is determined, only continuous states change before the next situation is played, and $S' \times C \rightarrow S'$. C triggers the generation of the next event tactics E. The event trigger function is $E = \varphi(C, R, t_E)$, where t_E is the event trigger time for E. Analogously, C is also the set of continuous schemes within the determined E, $C = \phi_E(X, Y, \alpha, \beta, J_E)$, where J_E is the objective function in $[t_{E,} t_{E+1}]$, and t_{E+1} is the first trigger time after t_E.

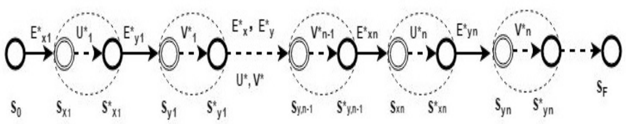

Fig. 2. Hybrid game evolution with optimal strategies (U^*, V^*) and best event tactics (E_x^*, E_y^*).

Fig.2 demonstrates a dynamic evolution path of a HDGs with (U^*, V^*, E_x^*, E_y^*), where

$$\left(S_0, S_{x1}, S_{x1}^*, S_{y1}, S_{y1}^*, \cdots, S_{xn}, S_{xn}^*, S_{yn}, S_{yn}^*, S_F^*\right) \quad (5)$$

is the node set, and E^* and C^* determines the path; ϕ_E and φ belong to the mapping functions from the above edge set to the node set. Then we obtain that

$$w = S_0 E_{x1}^* S_{x1} U_1^* S_{x1}^* E_{y1}^* S_{y1} V_1^* S_{y1}^* \cdots\cdots S_F$$

is a path with expected equilibrium (U^*, V^*, E_x^*, E_y^*)..

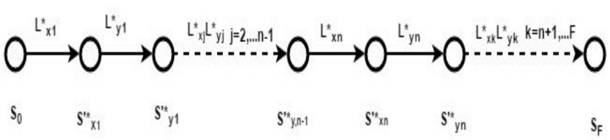

Fig. 3. Graph-based evolution path of HDG with expected equilibrium based on Fig.2

Fig. 3 illustrates a graph-based desired evolution path of a HDGs, where S_0 is the origin, S_F the destination, and S' the internal node set. $S_{pi}^{'*} \in S'$ has merged S_{pi} and $S_{pi}^*(p = \{x, y\}, i = 1, \cdots, n)$ as one dynamic node. $L*$ is the edge set and $L^* = \left(L_{x1}^*, L_{y1}^*, \cdots, L_{xi}^*, L_{yi}^*, \cdots, L_F^*\right)$. Letting $\Sigma^* : S^* \times L^* \rightarrow S^*$ be the mapping function, the expected path of the HDGs can be given as $w^* = \left(S_0 L_{x1}^* S_{x1}^* L_{y1}^* S_{y1}^* \cdots L_{xi}^* S_{xi}^* L_{yi}^* S_{yi}^* \cdots S_F^*\right)$. Then, the general networked model of HDGs is

$$G' = (S, L, \Sigma, R), \quad (6)$$

where $S = \{S_0, S', S_F\}$ is the set of all nodes and $S_{pi}^{'*} \in S'$ is represented by (1). L is the set of all edges, $L = (L_{x1},, L_{y1} \cdots, L_{xn}, L_{yn}, \cdots L_F)$. $\Sigma : S \times L \rightarrow S$ is the mapping function that shows the interaction between ϕ_E and φ, and it can be defined as the generation of E and C. Fig. 4 is a diagram of the networked model of a HDGs, which is a directed dynamic networks model.

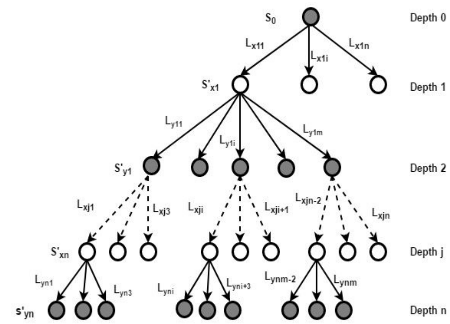

Fig. 4. Diagram of the directed networked model of HDGs.

IV. An Example

To show the effect of event-triggered moves on HDGs, we consider the dynamic games with a Lanchester $(1, 2)$ attrition model as

$$\left\{ \begin{array}{l} \dot{x}_1 = -\sum\limits_{j=1}^{2} \psi_{j1_k} \alpha_{j1} y_j + u_1 \\ \dot{y}_1 = -\beta_{11} \phi_{11_k} x_1 + v_1 \\ \dot{y}_2 = -\beta_{12} \phi_{12_k} x_1 + v_2 \end{array} \right. , \quad (7)$$

where $x_1(t)$, $y_1(t)$, and $y_2(t)$ are the strengths of two opposing players surviving at time t. The payoff function has the form

$$J = \eta_1 x_1 - \theta_1 y_1 - \theta_2 y_2. \quad (8)$$

Then all initial conditions can be assumed to be $x_{10} = 100, y_{10} = 30, y_{20} = 30$; $\alpha_{11} = 9, \alpha_{21} = 1, \beta_{11} = \beta_{12} = 1$ $T = 0.489$, $\eta_1 = 9, \theta_1 = 1, \theta_2 = 9$, $u_1 = 0, v_1 = v_2 = 0$. According to Propositions 1 and 2, it is easy to obtain that the trigger time is $\Delta_1 = 0.384$, and the best event move is

$$\left\{ \begin{array}{l} E_x^* = \left\{ \begin{array}{ll} \begin{bmatrix} 1 & 0 \end{bmatrix} & 0 < t \leq 0.348 \\ \begin{bmatrix} 0 & 1 \end{bmatrix} & 0.348 < t \leq 0.489 \end{array} \right. \\ E_y^* = \begin{bmatrix} 1 & 1 \end{bmatrix}, 0 < t \leq 0.489 \end{array} \right. .$$

Then we can get the event-triggered mechanism as

$$E = \{(E_{x1}, E_{y1}), (E_{x2}, E_{y2})\}, \quad (9)$$

where $E_{x1} = [1, 0]^T$, $E_{x2} = [0, 1]^T$, $E_{y1} = E_{y2} = [1, 1]^T$.

According to the above quantitative analysis, the graph-based extensive form of event-triggered dynamic games is

$$\Gamma = (S, L, \Sigma, R), \quad (10)$$

where $S = \{S_0, S_{x1}, S_{y1}, S_{x2}\}$ is the set of situations with invariable structures, which are completely presented by

$$\left\{ \begin{array}{l} \dot{x}_1 = -9y_1 \\ \dot{y}_1 = -x_1 \\ \dot{y}_2 = -x_1 \end{array} \right. , \left\{ \begin{array}{l} \dot{x}_1 = -y_2 \\ \dot{y}_1 = -x_1 \\ \dot{y}_2 = -x_1 \end{array} \right. , \quad (11)$$

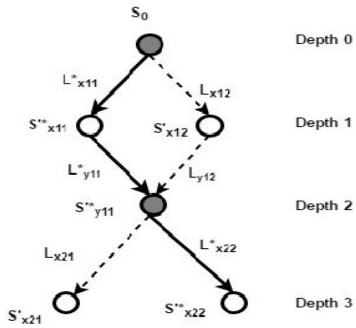

Fig. 5. Directed tree structure network model of one-to-two event-triggered dynamic games

L is the generation of E, and it can be proposed by $E = \{(E_{x1}, E_{y1}), (E_{x2}, E_{y2})\}$, $\Sigma : S \times L \to S$ the mapping function, and R satisfies the following rules

$$\psi_{j1_k} = 1, \quad \sum_{j=1}^{2} \phi_{ij_k} = 1. \tag{12}$$

Fig. 5 displays a graph-based evolution process for the above example, and a directed tree structure network model is given. Once R is certain, $\left\{S_0, S_{x11}^{'*}, S_{x12}^{'}, S_{y11}^{'*}, S_{x21}^{'}, S_{x22}^{'*}\right\}$ are all nodes that can be expressed by the continuous subsystem. $\left\{L_{x11}^{*}, L_{x12}, L_{y11}^{*}, L_{y12}, L_{x21}, L_{x22}^{*},\right\}$ is the edge set that reflects the change of all event tactics E. Σ is decided by the above nodes and edges. It should be noted that the directed solid lines depict the optimal evolution path

$$w^* = \left(S_0 L_{x11}^{*} S_{x11}^{'*} L_{y11}^{*} S_{y11}^{'*} L_{x22}^{*} S_{x22}^{'*}\right).$$

Because the proposed event moves can only be triggered once, we can get a two-layer temporal network model of the above one-to-two event-triggered dynamic games,

$$\Gamma' = (x, y, J, \Sigma, R), \tag{13}$$

where $x = x_1(t)$, $y = (y_1(t), y_2(t))$, and $J = (J_1, J_2)$ can be computed by considering $x_1(\Delta_1)$, $x_1(T)$, $y_1(\Delta_1)$, $y_2(\Delta_1)$, $y_1(T)$, $y_2(T)$, η_1, θ_1, and θ_2. Σ and R are similar to the corresponding elements of (7). Fig. 6 shows the diagram of a two-layer temporal network model of event-triggered dynamic games, where the number of layers is determined by the trigger times of event moves. It clearly illustrates the impact of the change of event moves on the evolution of the above game process, and the continuous dynamic changes of the game process are also well analyzed and presented.

Remark 2. For the above example, it is easy to know that Zeno behavior is avoided with the preset conditions of all parameters in Ref. [16]. The general conditions of avoiding Zeno behavior will be investigated in the future.

Remark 3. For the above example, we also find that the network evolution model of HDGs has the characteristics of bipartite graph. Even if the number of stages and branches of

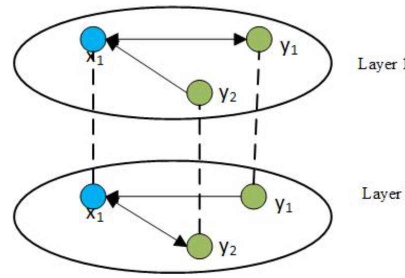

Fig. 6. Two-layer temporal network model of one-to-two event-triggered dynamic games

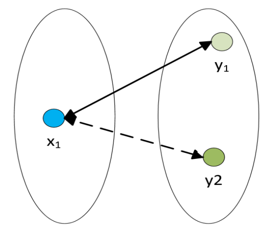

Fig. 7. Bipartite behavior of one-to-two event-triggered HDGs on each layer

both sides of the games are increasing, the bipartite graph characteristics of the established game network evolution model will not change as long as the cooperative behavior of each side is not considered. Fig.7 shows the bipartite graph behavior of the above example during evolution. Hence, when we set some reasonable assumptions, the bipartite network model is also an important model for the networked modeling of HDGs studied in this paper.

V. Conclusions and Future Works

By analyzing the dynamic evolution process of a HDGs, we focused on the networked modeling perspective regarding HDGs. On the basis of the proposed normal form of a HDGs, we established a directed dynamic network model for the first time. The basic framework is given in detail and an example of a HDGs reconstructed to explain the rationality of networked modeling. Planned future works include the equilibrium of a HDGs and self-triggered mechanisms will be considered. As an important application, How to propose the network-based model of a WHDGs can help enrich the theory of HDGs.

Because the emphasis of this paper is to give the basic network evolution model of HDGs, the analytical theory and solution technique of HDGs have not been studied. As a continuation of this study, possible future work includes the following.

(1) According to Remark 1, we easily get that the evolution of all event-triggered moves $(E_{x0}, E_{x1}, \cdots E_{xk}, \cdots)$ and $(E_{y0}, E_{y1}, \cdots E_{yk}, \cdots)$ shows dynamic characteristics. Due to $\psi_{ji_k} \in \{0, 1\}$ and $\phi_{ij_k} \in \{0, 1\}$, How to establish Boolean network-based model of HDGSS becomes the first problem to be solved in the next step.

(2) In view of the established HDGs network evolution model, when the system structure and complexity are increasing, we need to discuss its dynamic characteristics. On the one hand, there is cooperative behavior between y_1 and y_2. Especially, the cooperation emerges of the multi-layer temporal network model of HDGs with large strategy sets and multi-stages becomes the key to the follow-up research. On the other hand, we must first discuss the equilibrium of HDGs. In fact, when large strategy sets and multi-stages for HDGs exist, there are differences in the understanding of the action to ensure the maximization of interests between the two sides of the game, which leads to the situation being more important than the equilibrium. How to redefine the equilibrium needs to be resolved, and principles and possible methods of redefinition must also be discussed.

(3) According to the proposed network-based model of HDGs and some propositions, the evaluation of relatively important network nodes and the analysis of topology structure must be considered. This evaluation can help remove failing and further simplify the search space of large strategy sets. It would be helpful to design the solution theory and method of networked HDGs in the future. In addition, warfare hybrid dynamic games (WHDGs) and the evolutionary game of the supplies and demands of logistics are typical examples of HDGs. From the point of view of applications, how to propose a model and design the optimal schemes can help to enrich the theory of HDGs.

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